

# Proton Mass Mystery And Neutron Stars

*Now that the Higgs boson is discovered, does one know where the proton mass comes from ?*

Problem for “RAON” anticipated in 2000 at KIAS

**Mannque Rho**  
**CEA Saclay**

# Collaboration

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# The Question

❖ The proton mass is measured accurately

$$m_p = 938.27200 \pm 0.00004 \text{ MeV} \approx 1 \text{ GeV}$$

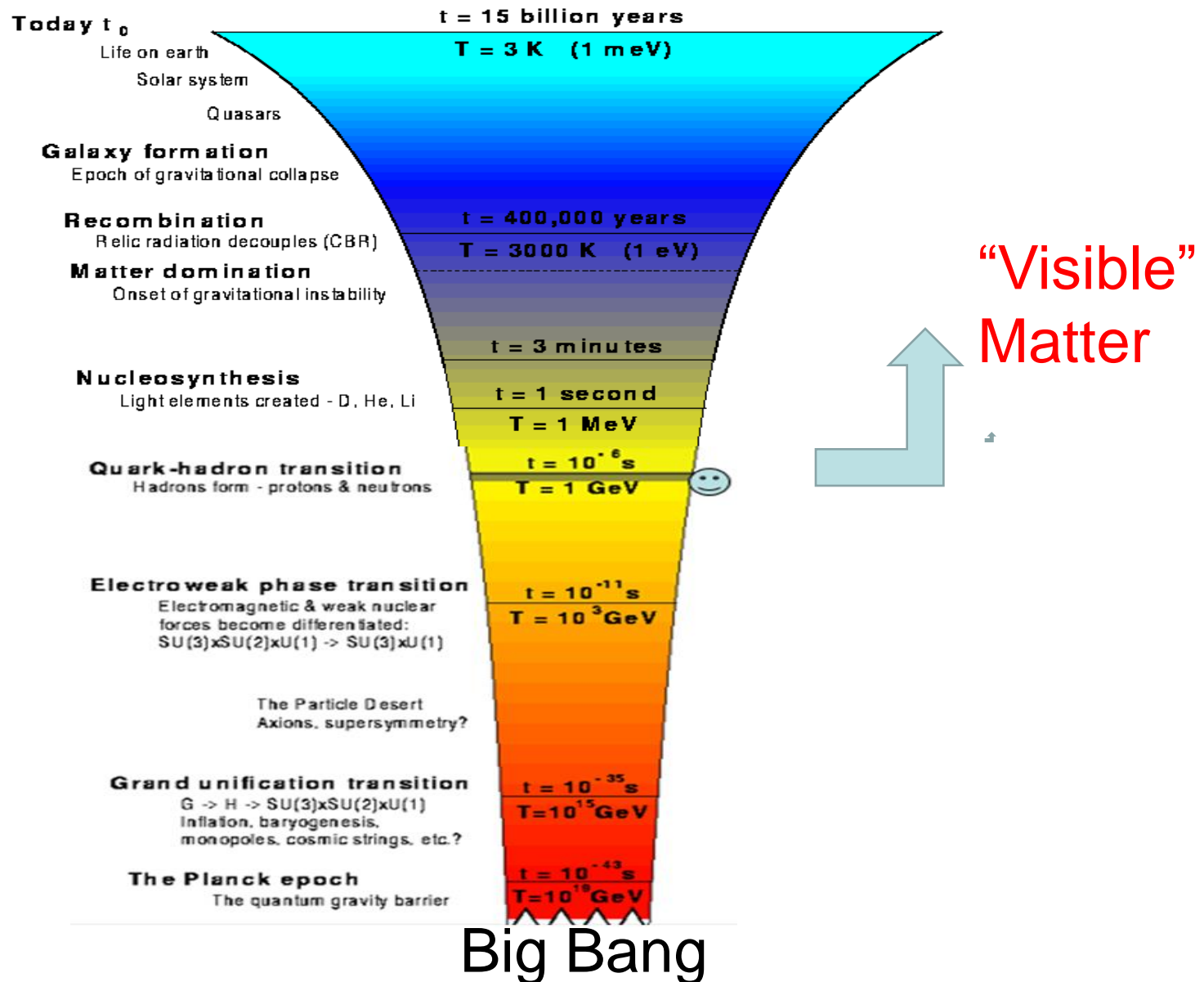
❖ The bulk (more than 90%) of the matter around us is made of

***protons and neutrons***

❖ But where does the proton mass come from??? Is  
THE BIG QUESTION

This MYSTERY is intimately  
Connected with  
the ***Early Universe***  
And  
***Super Dense Matter***  
as in compact stars

# Origin of Matter

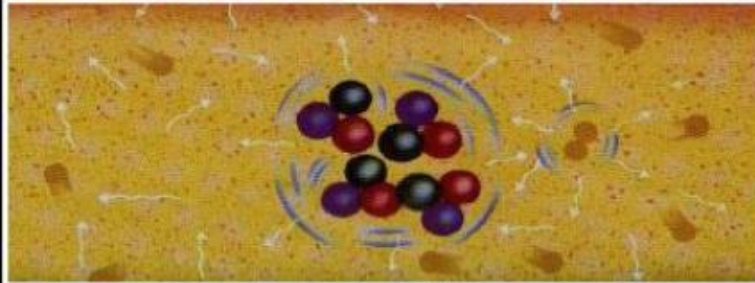
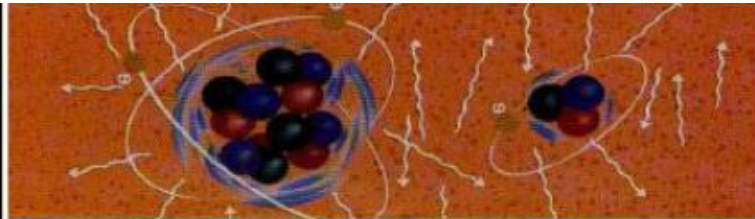


300,000  
years

3  
minutes

1 micro-  
second

1 pico-  
second



Origin of dark matter?



Origin of Matter?

**BANG!**

Formation  
of atoms

Formation  
of nuclei

Formation  
of protons  
& neutrons

Appearance  
of mass?

Proton mass  
Appears here

Superdense matter is  
fascinating .....

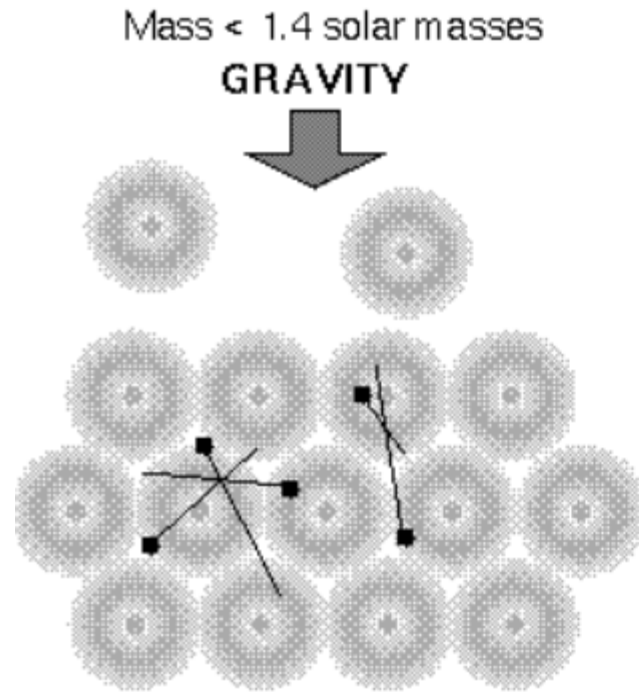
# Squash Manhattan to a teaspoonful



Or compress Boing 747 into a small grain of sand!!!



# Densest “visible” object in the Universe

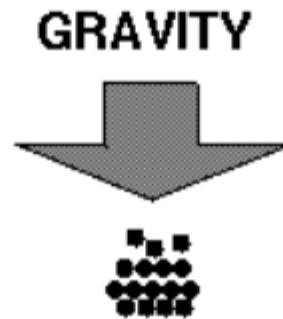


## White Dwarf

Electrons run out of room to move around. Electrons prevent further collapse. Protons & neutrons still free to move around.

Stronger gravity => more compact.

Mass > 1.4 solar masses  
but mass < 3 solar masses



## Neutron Star

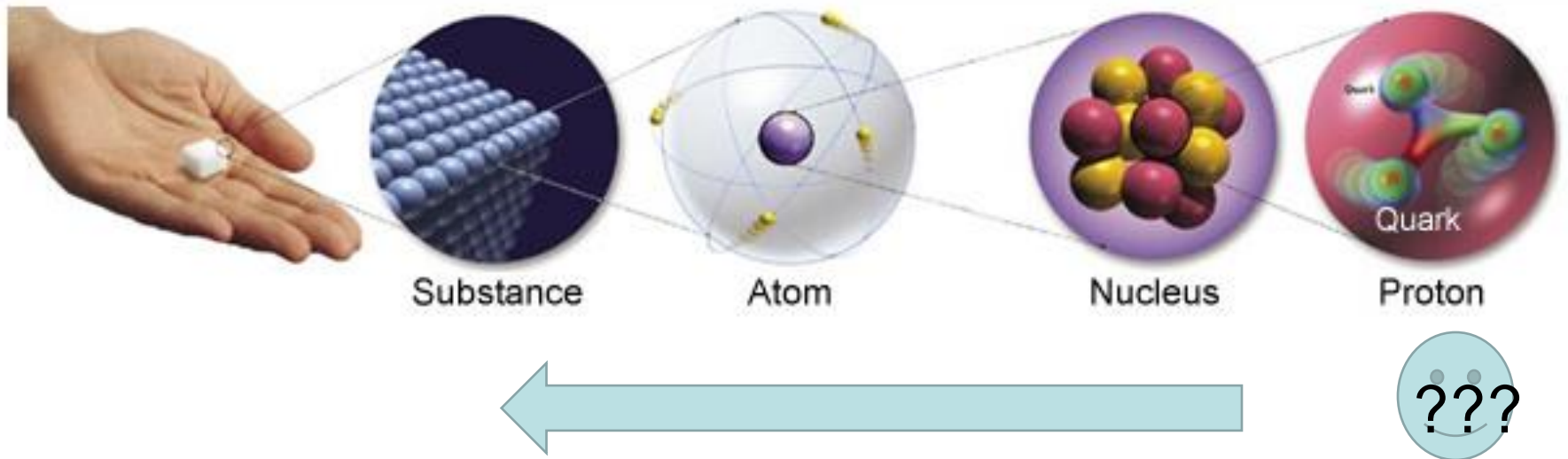
Electrons + protons combine to form neutrons. Neutrons run out of room to move around. Neutrons prevent further collapse. Much smaller!

Mass > 3 solar masses



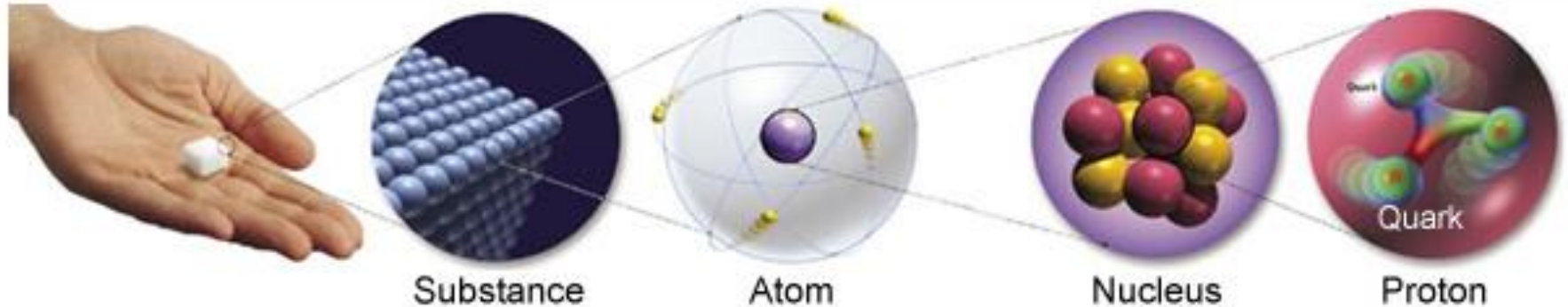
## Black Hole

Gravity wins!  
Nothing prevents collapse.



Chemistry ← molecular ← atomic  
← nuclear physics ← proton/neutron ← ??? ← quarks

# Stuff around us



- Proton mass:  $m_p = 938.272046 \pm 0.000021 \text{ MeV}$
- Lead 208 nucleus  $^{208}\text{Pb}$ :  $0.9926 \times 208m_p$

More than 99% of the mass of the lead nucleus ( $^{208}\text{Pb}$ ) with 208 nucleons (82 P, 126 N) comes from the simple sum of the nucleon (proton) mass.

$$\therefore \text{Mass of stuff around us} \\ = (0.99..) \times \text{number of nucleons} \times \text{proton mass}$$

# WikipediA

“For the proton, of mass  $m_p = 938$  MeV, the bound quarks only contribute about 10 MeV to its mass; the bulk of it arises out of **QCD chiral symmetry breaking**.”

“Yoichiro Nambu was awarded the 2008 Nobel prize in physics for his understanding of this phenomenon.”

**“Nambu mechanism:”**

“If chiral symmetry is made ‘unbroken’ the proton mass must then disappear”

## Nambu-Jona-Lasinio (1961)

... Our model Hamiltonian, though very simple, has been found to produce results which strongly simulate the general characteristics of real nucleons and mesons. It is quite appealing that both the **nucleon mass** and the pseudo-scalar '**pion**' are of the same dynamical origin, and the reason behind this can be easily understood in terms of (1) classical concepts such as attraction or repulsion between particles, and (2) the  $\gamma_5$  symmetry.

# Quark Masses

Light Quarks:  $q = u, d, s$        $m \equiv m_u \approx m_d \approx m_s \approx 0$



“Chiral symmetry”

*Relevant to “stuff” around us*

“QCDLite”

Heavy Quarks:  $Q = c, b, t$        $M \equiv m_c \approx m_b \approx m_t \approx \infty$



“Heavy-quark symmetry”

*Irrlevant to “stuff” around us*

# Puzzling

“We find that  $\sim 90\%$  of the proton (and neutron) mass, and therefore  $\sim 90\%$  of ordinary matter, emerges from an idealized theory – QCDLite -- whose ingredients are entirely massless.”

F, Wilczek 2012

A deep philosophical, not just physics, problem of “Mass without Mass” and “Something for Nothing”

$$m_P = \sum "??"$$

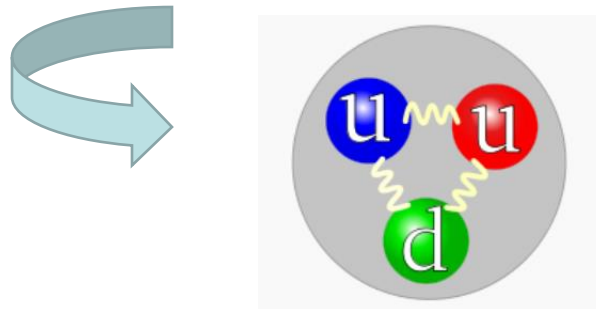
What is “??” ?

“?? Stuff”

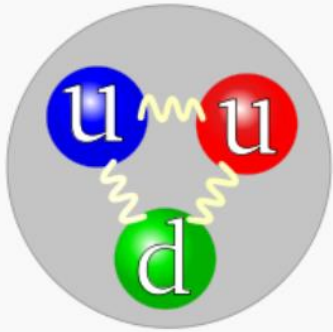
- Colored quarks:  $q, q, q$ ;  $q=(u,d,s, \dots)$
- Massless gluons

QCD says

Proton is made up of 3 (nearly) massless quarks (2u+1d) & massless gluons







$m_u \approx 2.5 \text{ MeV}$ ,  $m_d \approx 5.7 \text{ MeV}$   
from Higgs mechanism

Englert & Higgs 2013

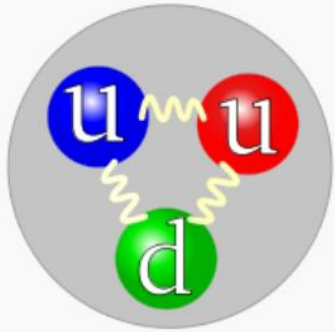
Naïvely

$$\begin{aligned} M_{\text{proton}} &= 2m_u + m_d + \text{“}\clubsuit\text{”} \\ &= 10.7 \text{ MeV} + \text{“}\clubsuit\text{”} \end{aligned}$$

cf. *Nature*  $\sim 1000 \text{ MeV}$

Sum of constituent (quark)  
masses does not work!

99% of the proton mass is  
*missing!*



## An interpretation of what QCD says

Up and down quarks are spin  $\frac{1}{2}$  fermions with tiny mass which can be ignored, so they satisfy massless **Dirac equation** interacting with exchange of massless gluons (“colored photons”)

### Confinement

Quarks are *NOT* seen at all, so must be “confined.”



# Story of Cheshire Cat

*Alice in the wonderland*

Lewis Carol



Quantum Cheshire Cat:  
Even Weirder Than Schrodinger's



# How QCD gives mass is

A story of chiral symmetry breaking by “jail”  
Or how the quarks escape from the “jail”

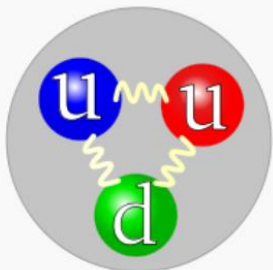
Massless quarks obey Dirac equation



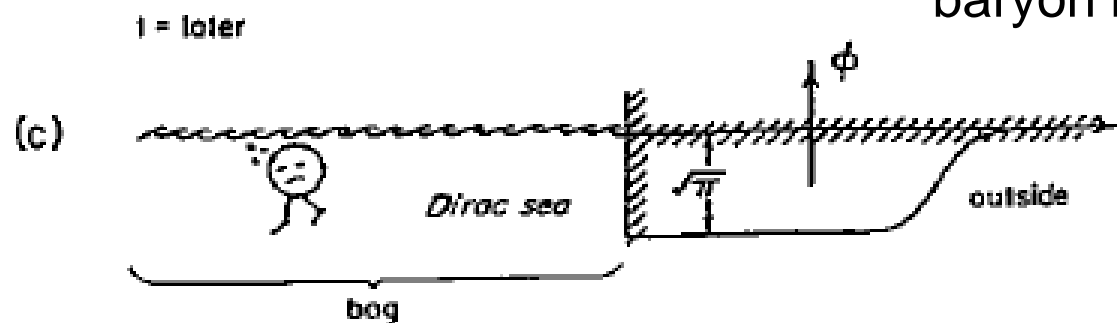
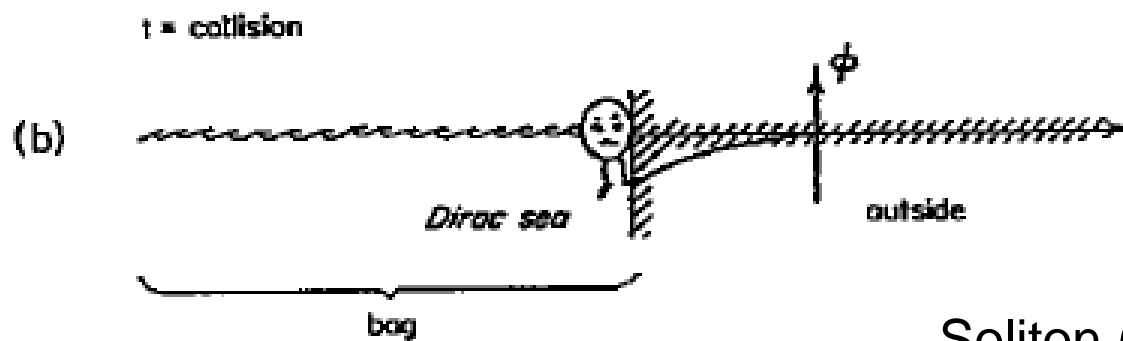
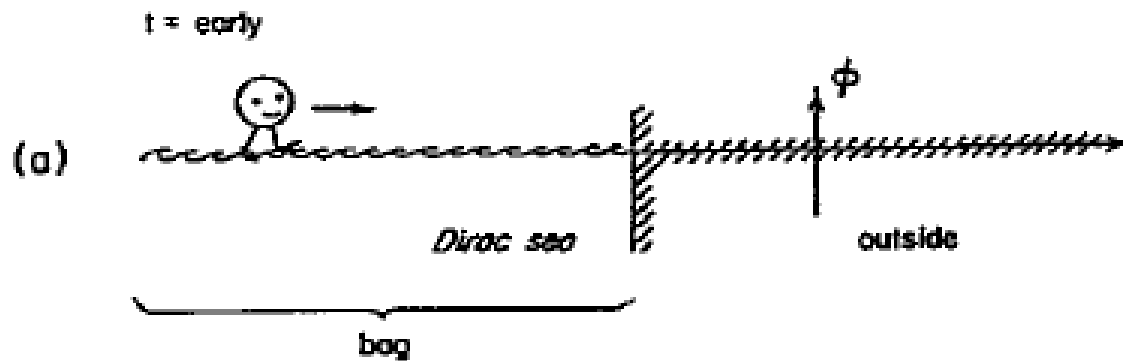
*Think in terms of one dimension*

- ❖ L(eftrightarrowt-going) quark must *always* go left
- ❖ R(ight-going) quark must *always* go right

Unless the “vacuum” blocks the path  
and breaks chiral symmetry



Courtesy  
H.B. Nielsen



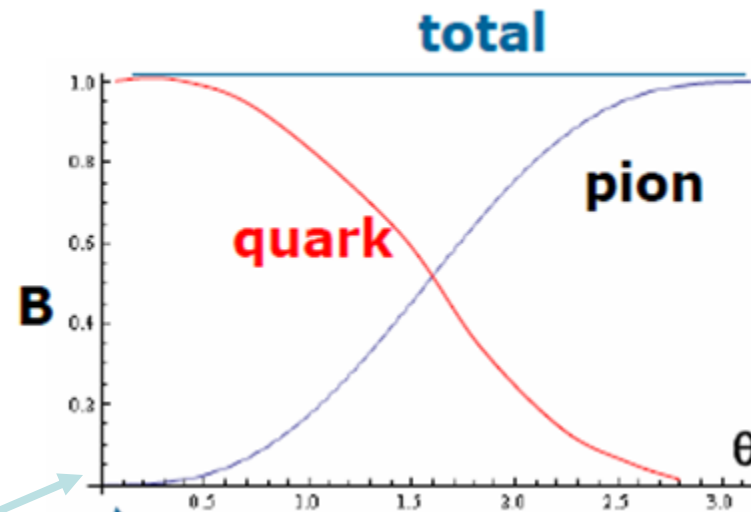
Soliton (skyrmion) carries  
baryon number

# In 4D

Baryon charge:

GEB, Goldhaber, R 1983  
Goldstone, Jaffe 1983

$$B = B_{out} + B_{in} = 1$$



$$L_{\text{QCD}} = \bar{\psi}(i\gamma_{\mu}D^{\mu} - m)\psi - \frac{1}{2}\text{Tr}G_{\mu\nu}G^{\mu\nu}$$

$$L_{\text{EFT}} = \frac{f_{\pi}^2}{4}\text{Tr}(\partial_{\mu}U\partial^{\mu}U^{\dagger}) + \dots$$

# Color anomaly

Nielsen, Wirzba, Zahed, R. 1991

- **Classically** color is “confined” inside the bag

$$\hat{n} \cdot \mathbf{E}^a = 0, \quad \hat{n} \times \mathbf{B}^a = 0.$$

- But  $\eta'$  coupled to quarks at the surface induces **quantum** anomaly and deposits color charge at the surface.
- Color charge leaks out into the meson sector and violate *color gauge invariance*, so *must* be stopped by the gauge-noninvariant surface **classical** counter term

$$S_{CT} = -\frac{g_c^2 N_F}{8\pi^2} \int_{\partial V} d\beta A_0^a \hat{n} \cdot \mathbf{B}^a \eta' / f_0.$$

→ Restore gauge invariance at the **quantum** level.

(cf. Green-Schwarz : First string revolution)

## → Cheshire Cat Action

$$S = S_V + S_{\bar{V}} + S_{\partial V},$$

$$S_V = \int_V d^4x \left( \bar{\psi} i \not{D} \psi - \frac{1}{2g_c^2} \text{tr} G_{\mu\nu} G^{\mu\nu} \right)$$

$$S_{\bar{V}} = \frac{f^2}{4} \int_{\bar{V}} d^4x \left( \text{Tr} \partial_\mu U^\dagger \partial^\mu U + \frac{1}{4N_f} m_{\eta'}^2 (\text{Tr} \ln U - \text{Tr} \ln U^\dagger)^2 + \dots \right)$$

$$S_{\partial V} = \frac{1}{2} \int_{\partial V} d\Sigma^\mu \left\{ (n_\mu \bar{\psi} U \gamma^5 \psi) + i \frac{g_c^2}{16\pi^2} K_{5\mu} (\text{Tr} \ln U^\dagger - \text{Tr} \ln U) + \dots \right\}$$

Color anomaly cancelled by the surface term

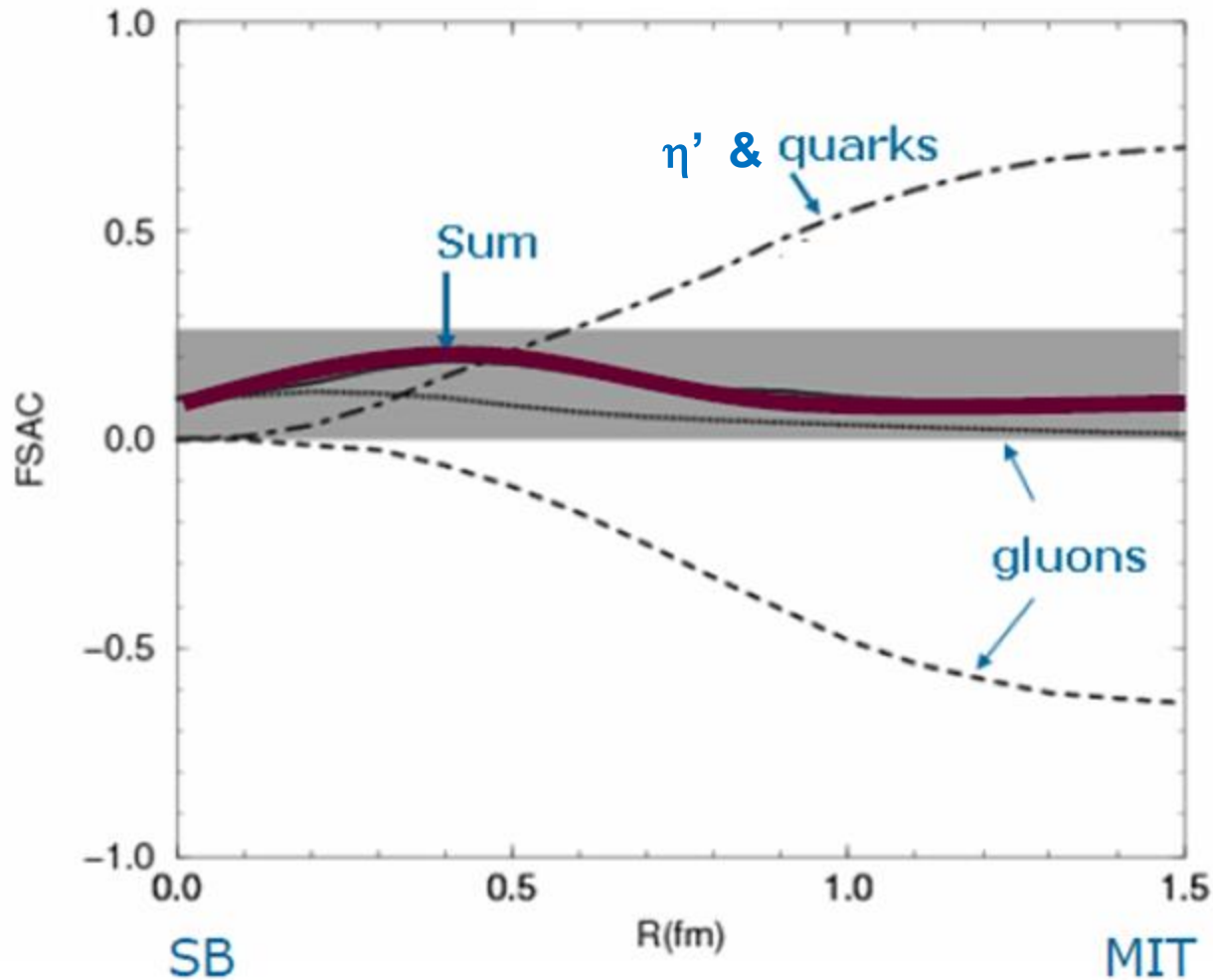
In the “chiral-bag gauge,” physics should not depend on the bag radius R



# FSAC (flavor singlet axial charge: $g_A^0$ )

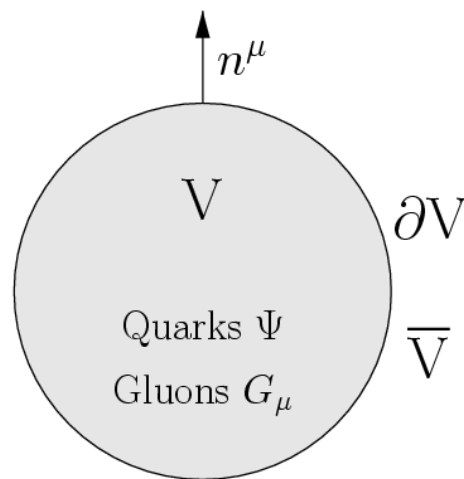
Hee-Jung Lee, Byung-Yoon Park, Min, Vento, R. 1999

How Cheshire Cat works!



FSAC has nothing to do with the proton spin!!!

# Cheshire Cat Phenomenon (CCP)



Goldstone bosons ( $\pi$ )

$\eta'$

**Pion cloud**

- **Baryon charge anomaly**, leaked charge carried by skyrmion.
  - **Color charge anomaly** (quantum), leaking stopped by surface (classical) counter term.
  - **Wess-Zumino terms** match inside and outside.
- $\therefore$  Bag radius is a *“gauge artifact”*.

## “Cheshire Cat” is a Gauge Theory

“Bag” is a *gauge artifact*, hence physics should not depend on it.

A proof in (1+1)D Damgaard, Nielsen and Sollacher 1994

Gauge condition:  $\Phi[\theta, \chi, \bar{\chi}] = 0$

$$\begin{aligned}\delta(\Phi[\theta, \bar{\chi}, \chi]) &= \int [db] e^{i \int d^2x b \Phi} \\ &= \int [db] e^{i \int d^2x \mathcal{L}_{g.f.}}\end{aligned}$$

Faddeev-Popov

$$\delta_\alpha \mathcal{L}_{g.f.} = -\frac{1}{\pi} \{ (1 - \Delta) + \Delta \} \alpha$$

“Chiral bag gauge”  $\Delta(x) = \Theta(x - z) \quad z = \hat{r}R$

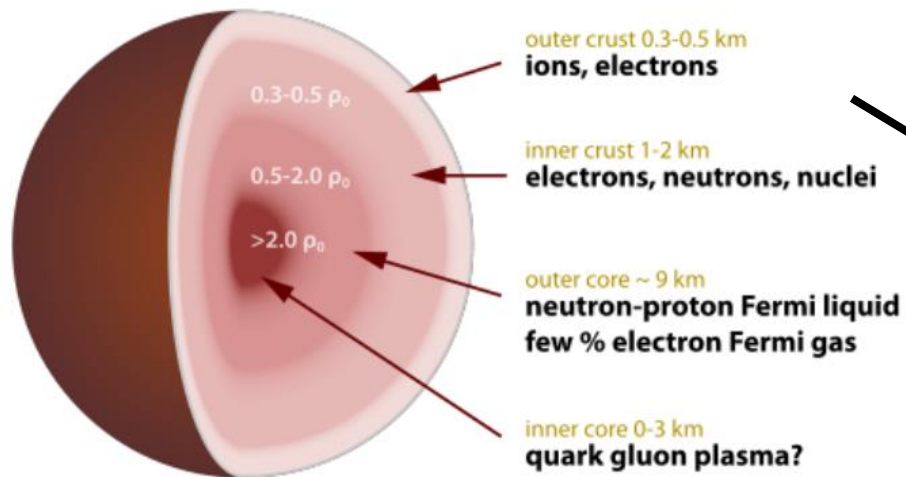
# How to “unbreak” chiral symmetry

Tweak  $\Sigma \neq 0 \rightarrow \Sigma = 0$  by compressing matter to  $\sim 10$  times nuclear matter density

- In laboratory (strong interactions)
- In space (gravity)

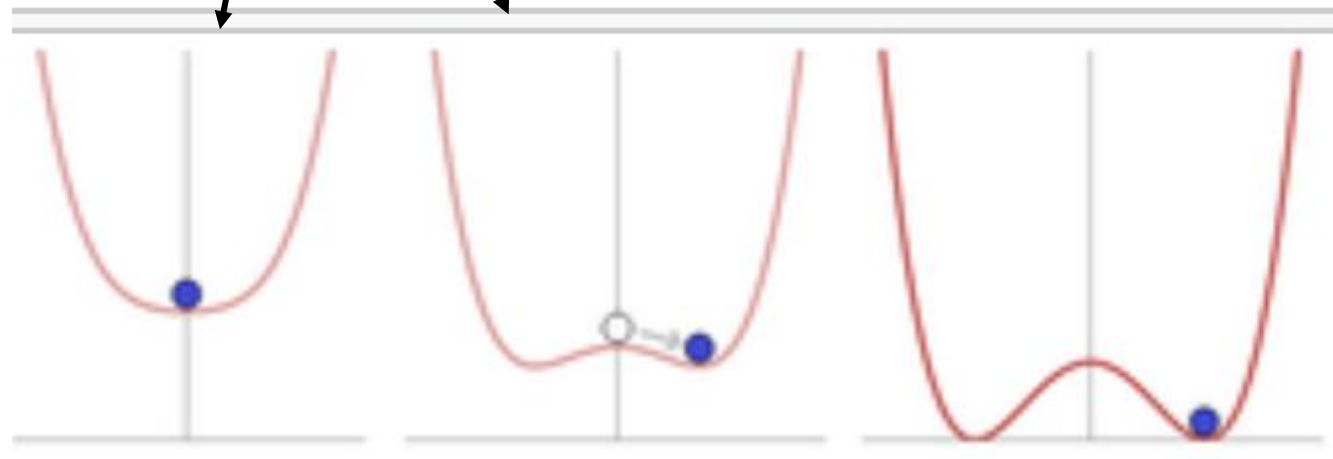


Neutron stars can decipher the mystery



Gravity At Work

Potential  $V$



Condensate  $\Sigma$

$= 0$

$\neq 0$

$\neq 0$

Density


$> 4 \rho_0$

$\approx \rho_0$

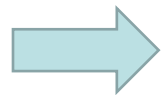
$< \rho_0$

Density increase

# Dense Matter Meets Gravity

- 
- Dense matter: solve **QCD** → **EoS** (Unknown)  
(Pressure vs. energy density)
  - Gravity: **Einstein Equation** (Known)

**TOV** (Tolman-Oppenheimer-Volkov) **Equation**



Stable compact stars (+ Black Holes)

*Mass ( $M$ ) vs. Radius ( $R$ ) etc. ....*

Nature 467, 1081 (2010)

# A two-solar-mass neutron star measured using Shapiro delay

**J1614-2230**  $1.97 \pm 0.04 M_{\odot}$

P. B. Demorest<sup>1</sup>, T. Pennucci<sup>2</sup>, S. M. Ransom<sup>1</sup>, M. S. E. Roberts<sup>3</sup> & J. W. T. Hessels<sup>4,5</sup>

Science 340, 1233232 (2013)

# A Massive Pulsar in a Compact Relativistic Binary

**J0348+0432**  $2.01 \pm 0.04 M_{\odot}$

John Antoniadis,\* Paulo C. C. Freire, Norbert Wex, Thomas M. Tauris, Ryan S. Lynch, Marten H. van Kerkwijk, Michael Kramer, Cees Bassa, Vik S. Dhillon, Thomas Driebe, Jason W. T. Hessels, Victoria M. Kaspi, Vladislav I. Kondratiev, Norbert Langer, Thomas R. Marsh, Maura A. McLaughlin, Timothy T. Pennucci, Scott M. Ransom, Ingrid H. Stairs, Joeri van Leeuwen, Joris P. W. Verbiest, David G. Whelan

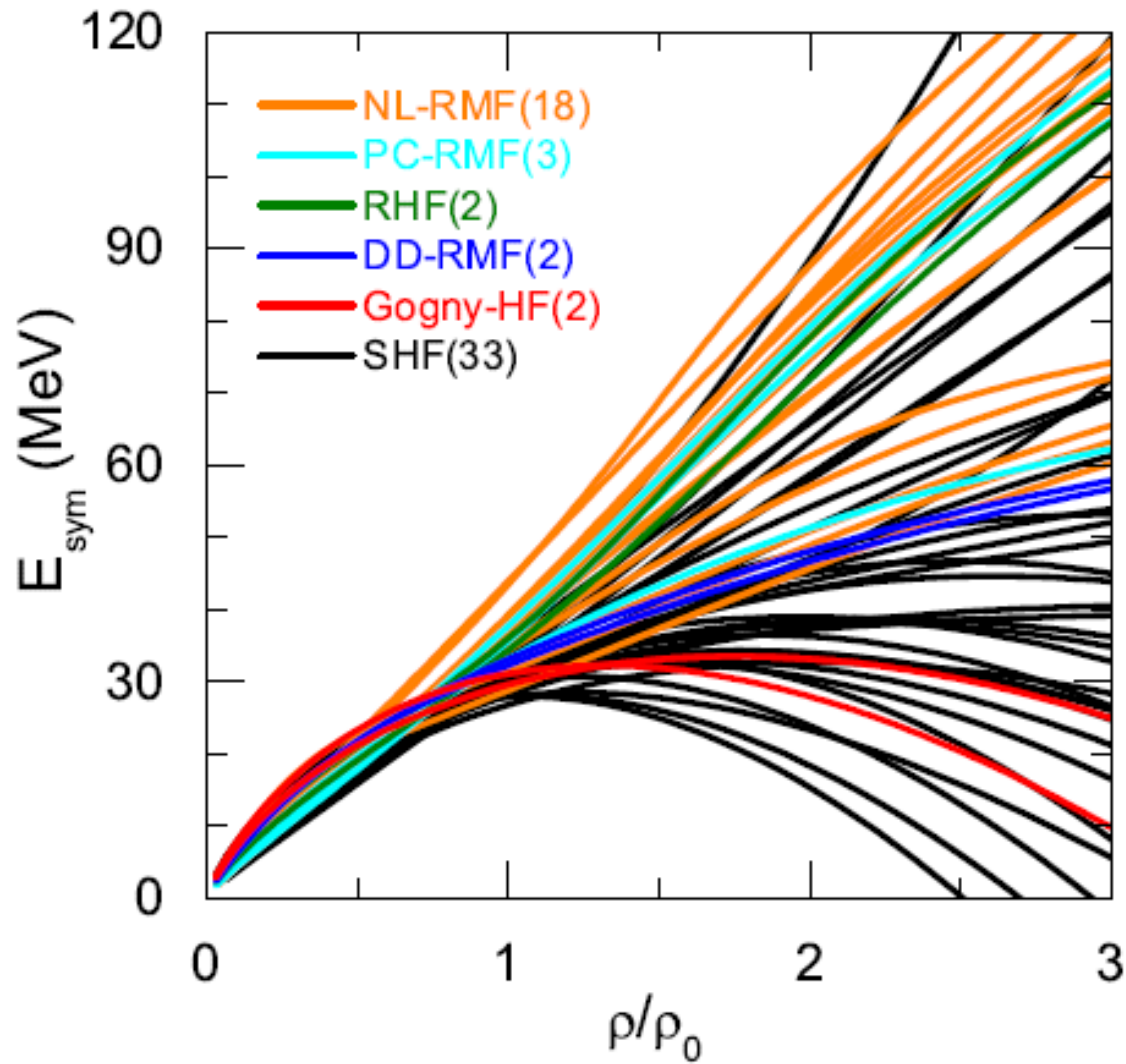


# Challenge: Dense Matter

“Mass gap” in QCD remains unsolved, so the EoS is not (mathematically) known.

This is the famous “Clay Millenium Math Problem”

We don't understand!!



# Two Laws of Physicists

Tsung-Dao Lee

❖ First law:

*Without experimentalists, theorists tend to drift.*

❖ Second law:

*Without theorists, experimentalists tend to falter.*

For mathematicians

*One-million \$ Clay Millenium problem*

Let mathematicians solve  
“Clay Millenium Math Problem”

Physicists resort to T.D. Lee's law:  
Guess EoS from Nature (experiments)  
→ Effective field theory (EFT)

*What makes physicists different from  
mathematicians!*

# Weinberg Folk Theorem

*“What is quantum field theory, and what did we think it is?” 1997*

“When you use quantum field theory to study low-energy phenomena, then according to the folk theorem you’re not really making any assumption that could be wrong, unless of course Lorentz invariance or quantum mechanics or cluster decomposition is wrong, provided you don’t say specifically what the Lagrangian is. As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you’re simply writing down the most general theory you could possibly write down.”

**Folk Proof:** “It’s hard to see how it can go wrong...”

# From Topology: Skyrmions

Atiyah and Manton arXiv: 1609.02816

We propose a new geometrical model of matter, in which neutral atoms are modelled by compact, complex algebraic surfaces. Proton and neutron numbers are determined by a surface's Chern numbers. Equivalently, they are determined by combinations of the Hodge numbers, or the Betti numbers. Geometrical constraints on algebraic surfaces allow just a finite range of neutron numbers for a given proton number. This range encompasses the known isotopes.

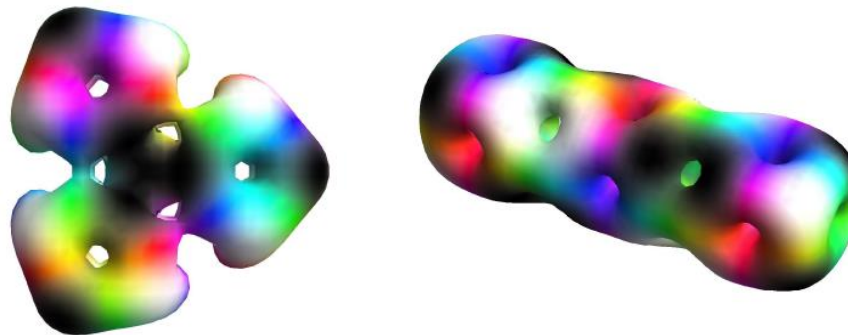
# In the limit $N_c \rightarrow \infty$ in QCD, baryons as skyrmions: Hoyle states in C-12

Lau & Manton 2014

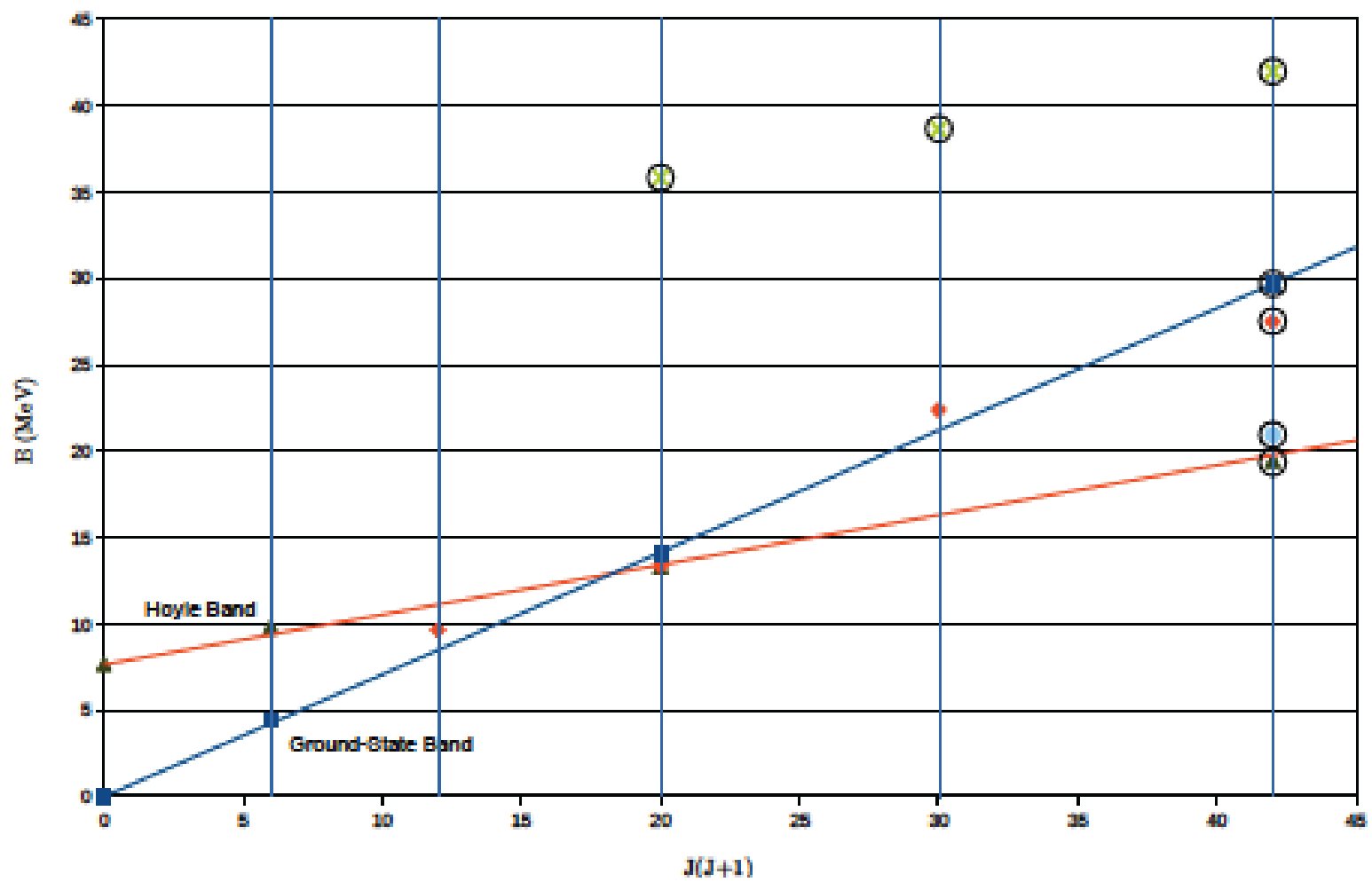
$$L = \int \left\{ -\frac{1}{2} \text{Tr} (R_\mu R^\mu) + \frac{1}{16} \text{Tr} ([R_\mu, R_\nu][R^\mu, R^\nu]) + m^2 \text{Tr} (U - 1_2) \right\} d^3x$$

$$R_\mu = (\partial_\mu U) U^\dagger, \text{ **Only pions**}$$

Treat to  $O(1/N_c)$  à la rotational quantization and generate rotational states. Two symmetries  $D_{3h}$  and  $D_{4h}$  for C-12



Only one parameter:  $m$  (pion mass)





## Hidden Local (Gauge) Symmetry

$$U = e^{2i\pi/f_\pi} = \xi_L \xi_R^\dagger, \quad \xi_{L,R} \in SU(2)_{L,R}$$

$$\xi_{L,R} \rightarrow \xi_{L,R} h^\dagger(x) \quad h(x) \in SU(2)_{L+R}$$

Elevate to gauge symmetry → “hidden” local symmetry  
→ Vector meson ( $\rho$ ,  $\omega$ ,  $a_1$ ) scale

Bando, Kugo, Yamawaki 1988

H. Georgi 1989

Harada, Yamawaki 2003

$$\mathcal{L}_{hls} = \frac{f_\pi^2}{2} \{ \text{Tr}(D_\mu \xi_L)^2 + \text{Tr}(D_\mu \xi_R)^2 \} + \kappa \text{Tr}(D_\mu U)^2$$



Spontaneous SB

## Infinite tower by “moose” construction à la Georgi

$$\rho, \rho', \dots, \omega, \omega', \dots, a_1 \dots$$

$$U(x) = e^{2i\pi/f_\pi} = \Sigma_0 \Sigma_1 \Sigma_2 \bullet \bullet \bullet \Sigma_\infty$$

❖ 5-Dimensionally deconstructed QCD (Son & Stephanov 04)

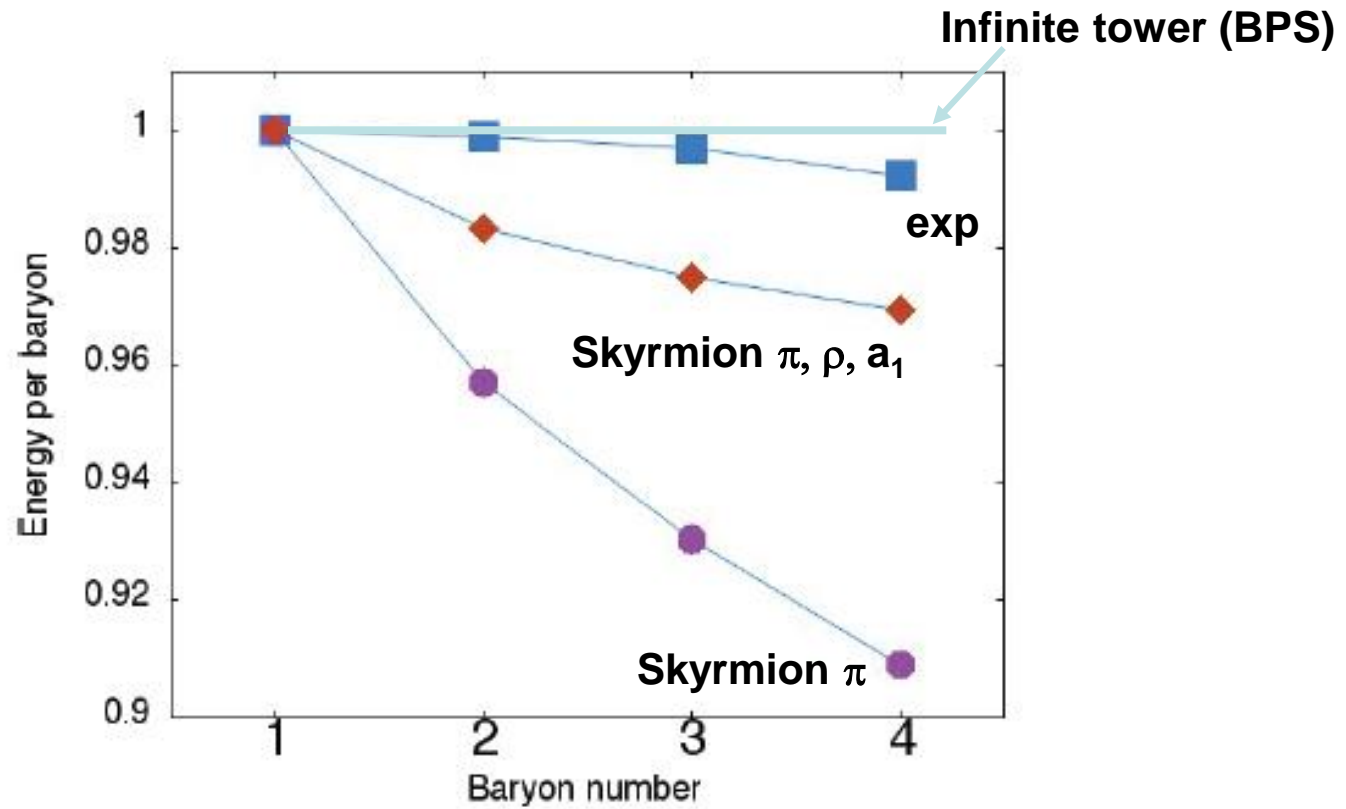
$$S = \int d^4x dz \frac{-1}{2g(z)^2} \sqrt{g} \text{Tr}(F_{AB} F^{AB}) + \bullet \bullet \bullet$$
$$A, B = 0, 1, 2, 3, z$$

Descends from string theory to holographic QCD !

Sakai & Sugimoto 2005

# Working of vector mesons

Sutcliffe 2011



# Infinite Tower of Vector Mesons → BPS Nuclei

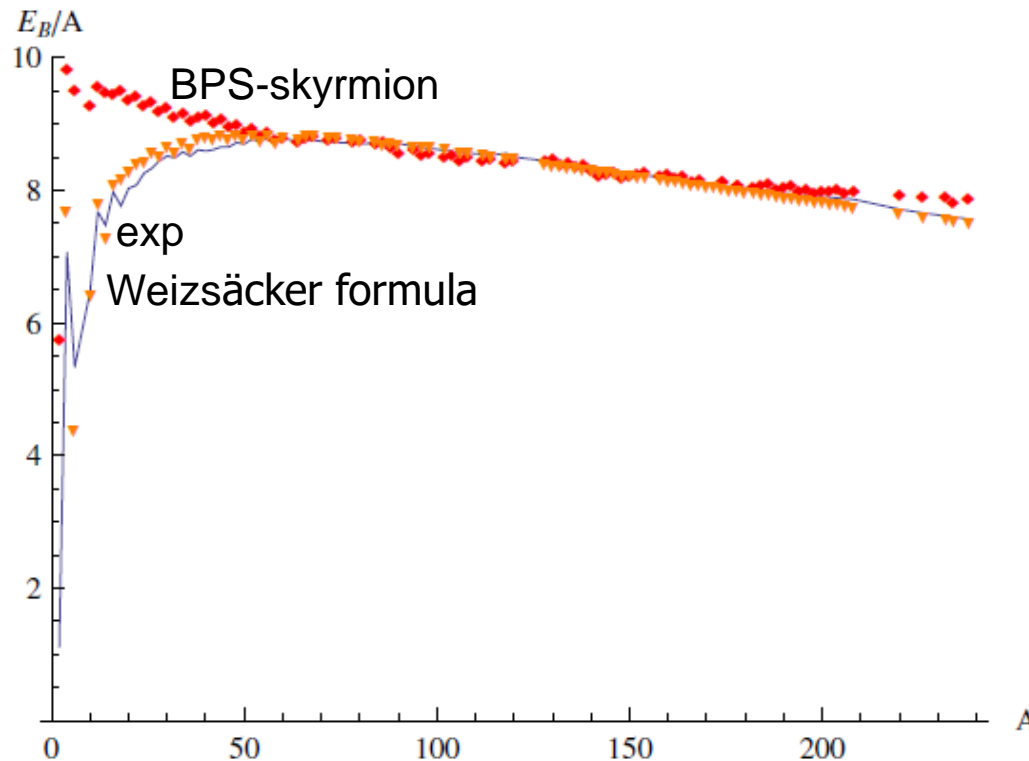
“BPS” nuclei are gotten when the space is flat  
&  $N_c$  &  $\lambda$  ('t Hooft constant)  $\rightarrow \infty$  !!

# BPS skyrmions

Adam, Naya, Sanchez-Guillen & Wereszynski 2013

BPS skyrmion matter is relativistic non-barotropic perfect fluid.

With 3 parameters and a small correction from Coulomb and isospin breaking.



And compact stars?

# Hidden Scale Symmetry: Story of “ $\sigma$ ” $\approx f_0$ (500)

“Scale invariance is ‘hidden’ in linear sigma model or NJL model”

K. Yamawaki, arXiv:1605.01951

Gell-Mann-Levy linear  $\sigma$  model

$$\mathcal{L}_{L\sigma} = \frac{1}{2} [(\partial_\mu \hat{\sigma})^2 + (\partial_\mu \hat{\pi}_a)^2] - \frac{\hat{\mu}^2}{2} (\hat{\sigma}^2 + \hat{\pi}_a^2) - \frac{\lambda}{4} (\hat{\sigma}^2 + \hat{\pi}_a^2)^2$$

Redefine fields

$$\mathcal{L}_{L\sigma} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{4} \sigma^2 \cdot \text{tr} (\partial_\mu U \partial^\mu U^\dagger) - \frac{\lambda}{4} (\sigma^2 - f^2)^2$$

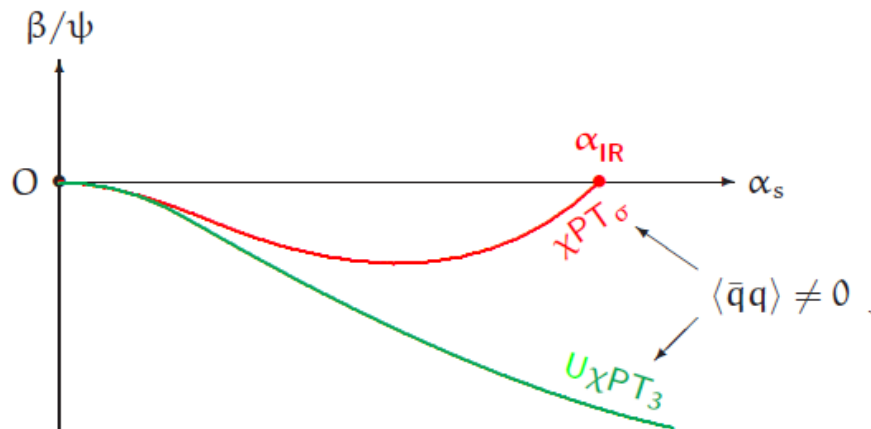
- (a) Strong coupling  $\rightarrow$  nonlinear sigma model, no scalar, ChPT etc.
- (b) Weak coupling  $\rightarrow$  scale invariance with dilaton  $\chi$ , **absent in medium-free QCD** due to trace anomaly

Density probes from (a) to (b)

Scale symmetry may emerge in medium.

## Daring Idea: IR fixed point ( $\beta=0$ )

IR fixed point with  $f_\phi = f_\pi$  in QCD for  $N_F=3 \rightarrow \phi=f_0(500)$   
 with  $\langle\chi\rangle = \langle\bar{q}q\rangle \neq 0$  Crewther and Tunstall 2015



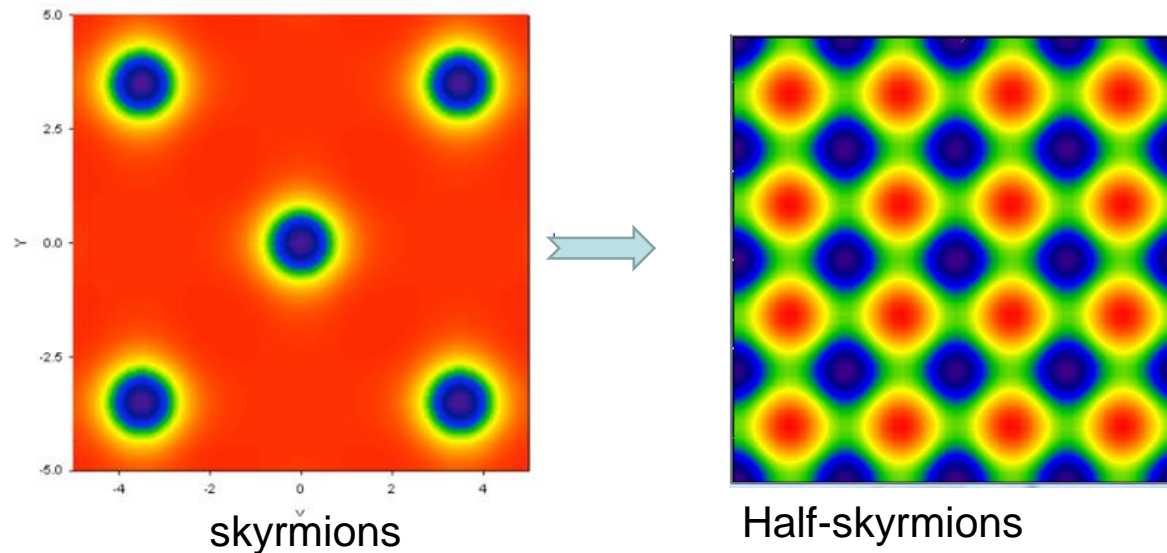
$$\theta_\mu^\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^2 + (1 + \gamma_m) \sum_{q=u,d,s} m_q \bar{q}q$$

$\Rightarrow$  NG bosons  $\pi, K, \eta, \sigma$ .

## Topology Change

At  $n=n_{1/2}$  skyrmions fractionize to half-skyrmions  
This is a **robust** prediction

Goldhaber/Manton, Gudnason/Nitta





# Scenario for $n > n_{-1/2}$

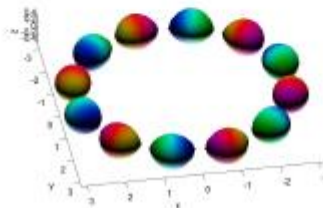
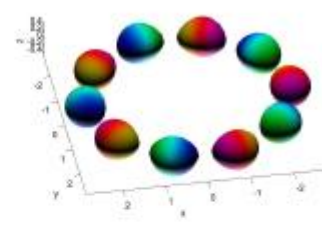
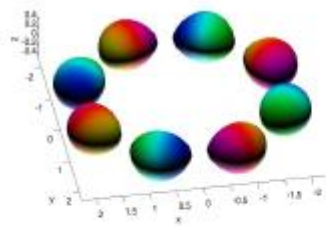
Gudnason and Nitta, PRD 91 (2015)

Break chiral symmetry explicitly

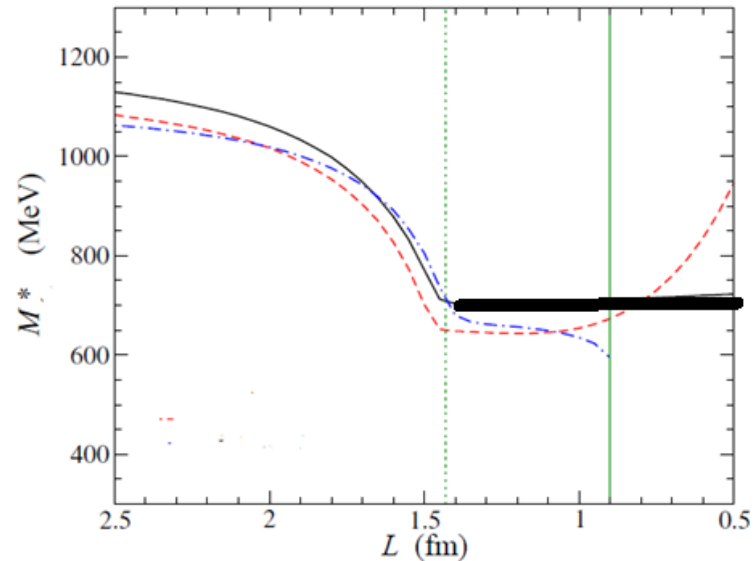
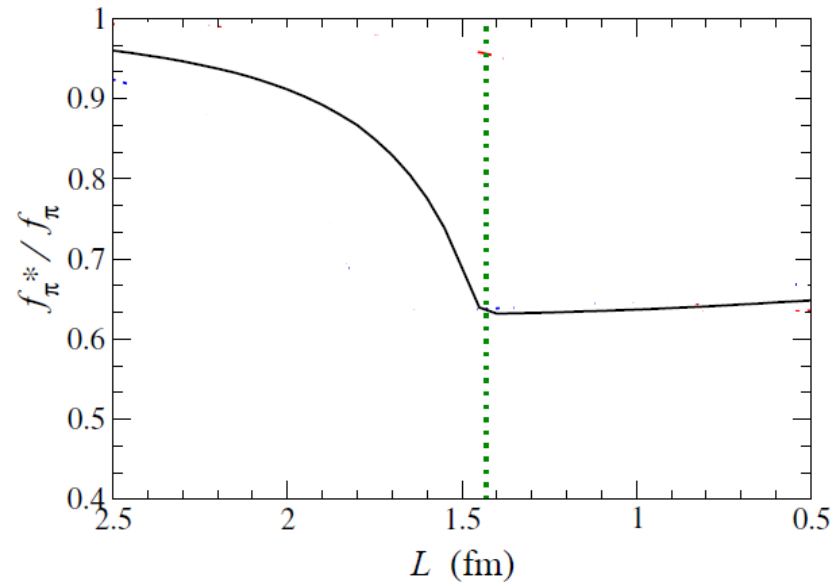
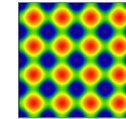
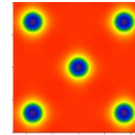
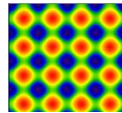
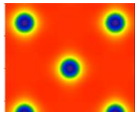
$$SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R} \rightarrow O(2)$$

$$V(U) = m^2 n_4^2, \quad U \equiv \mathbf{n} \cdot \mathbf{t}$$

Half-skyrmion molecules  $A=4,5,6$



# Topology change: Skyrmions $\rightarrow$ $\frac{1}{2}$ -skyrmions in dense matter



$$m_N \rightarrow \sim 800 \text{ MeV}$$

as  $\Sigma \rightarrow 0$ .

*Where the mass comes from???*

**Skyrmion** is a “**magnetic monopole**”  
in “hidden  $U(1)_\zeta$  symmetry”

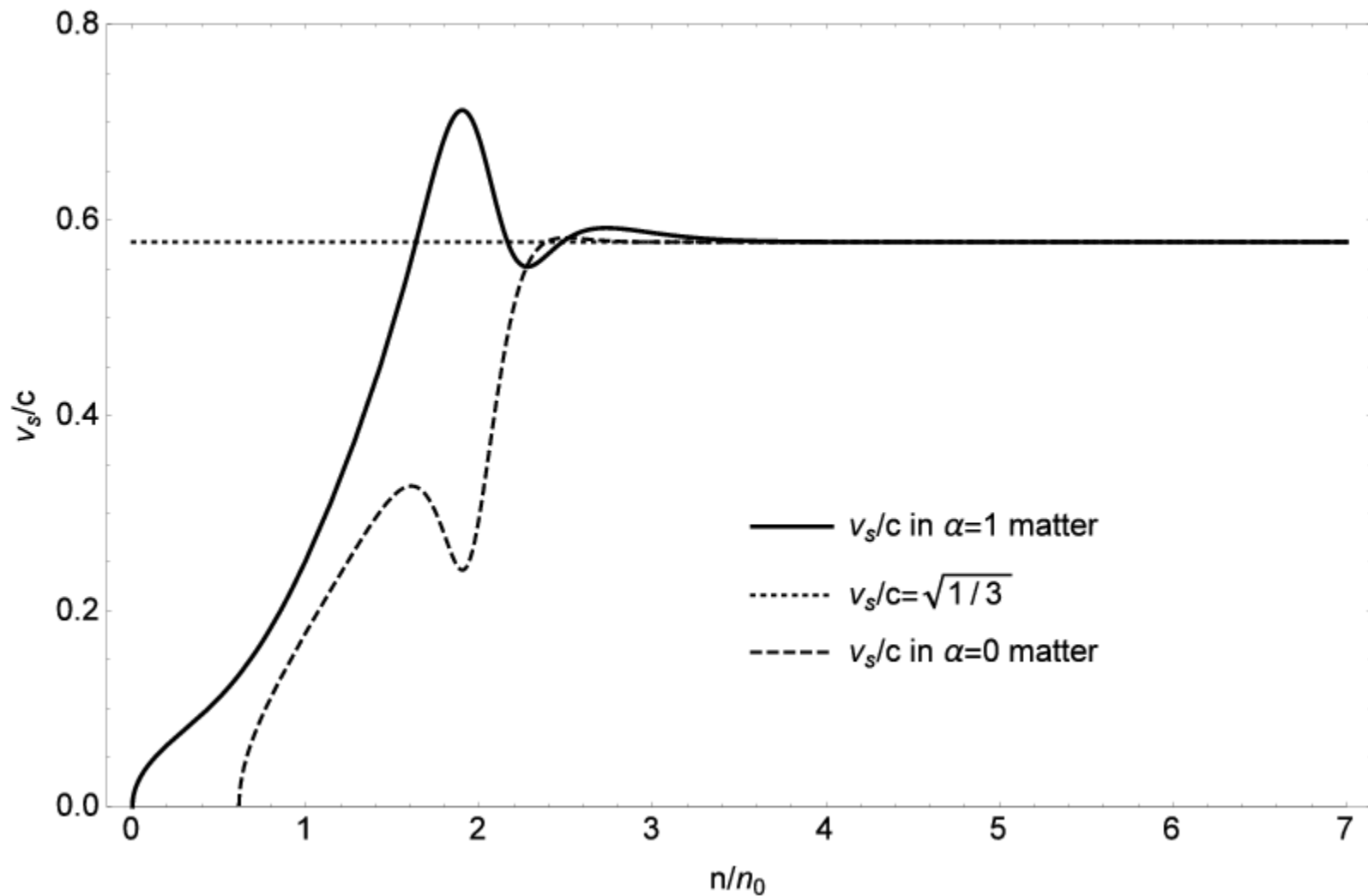
For  $n < 2n_0$ . skyrmion matter = *Fermi liquid pierced with “monopole” flux* and for  $n > 2n_0$ , half-skyrmion matter = *Fermi liquid pierced with “meron” or “dyon” flux*.

$$(v_s/c)^2 \rightarrow 1/3 \text{ and } \theta_\mu^\mu = \partial_\mu D^\mu = 0 \text{ with } f_\phi = f_\pi \neq 0.$$

*Emergent symmetry!*

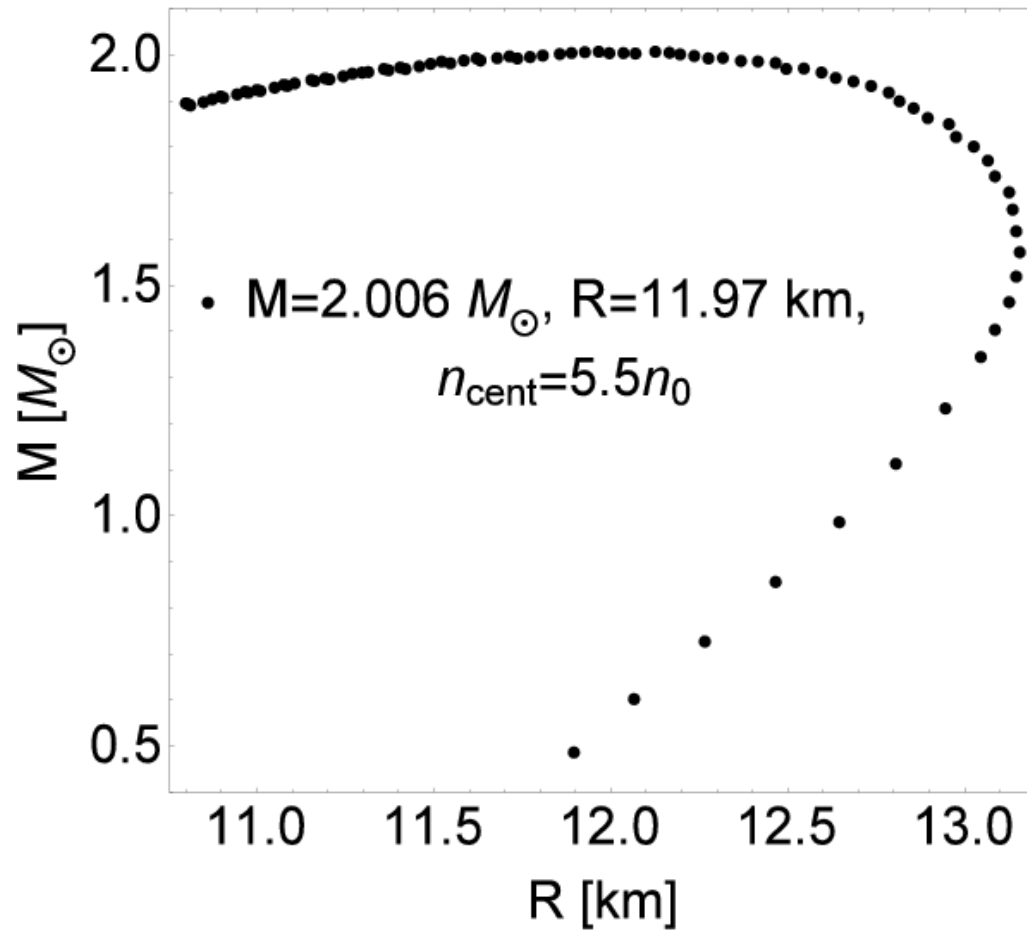
# Precursor to *conformal invariance*!(?)

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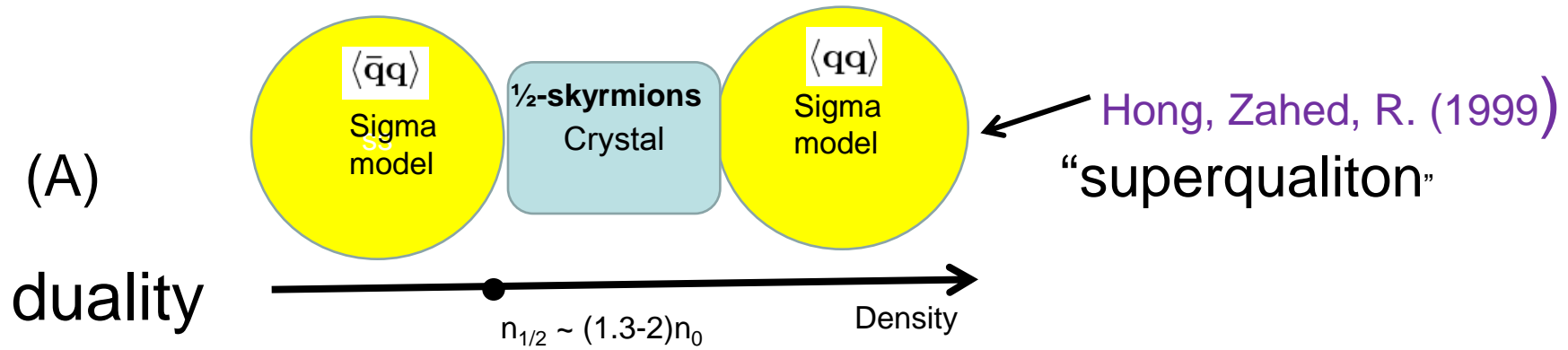


# TOV

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# Conclusion: Cheshire Cat at work Deconfined quantum critical?



(B) Baryons  $\rightarrow$  Quarkyonic  $\rightarrow$  CFL  $\leftarrow$  Fukushima, Kojo 2016

